Systems and Laws

* 1. **Simple Dynamical Systems and the state-space**

In Physics, generally we talk about Systems. From simple systems with a single particle that obeys a simple dynamical law, up to complex systems in multiple dimensions with an infinite amount of particles, or N-Particles.

If you know everything about a system at some instant of time, and you know the equations that govern how the systems changes, then you can predict its future or past state. This is a feature of all classical systems. (deterministic and reversible)

Every Classical Laws of Physics and every classical system are reversible and deterministic!

**System:** A collection of objects, particles, fields, waves, etc., that behave according to a set of equations.

**Closed System:** A system that is either the entire universe or an isolated, from everything else systems that behave as if nothing else exists.

**Open System:** A system that has external interactions with other systems.

**State-Space:** The collection of all possible states occupied by a system.

For example, let's say you have a closed system with a single coin glued on a table showing Heads. The state-space of the coin is equal to 1, since, it can occupy only 1 possible state, namely Heads.

**The State Space** is a mathematical set, whose elements label the possible states of the system.

The state-space of our example consists of a single point, as said before. We can predict its future state with extreme ease, since nothing ever happens that can change the outcome of our observation of the object’s state.

**Continuous Systems:** The Systems that evolve smoothly, without any discrete jumps or interruptions.

**Stroboscopic Systems:** The systems that evolve in discrete steps labeled by integers.

**Dynamical Systems:** The systems that change as time passes.

Dynamical Systems don’t consist only of a state-space, but they also entail a dynamical law of motion.

**Dynamical Law:** A law that tells us the next state of a system, given the current.

You can think of it as a function of time that has as its input the current state of the system, and outputs the next state.

The simplest example of a dynamical law is a loop. Whatever the current state is, the next state will be the same.

Let's say we have a coin so that :

T

H

This means that after the first observation, all others will be the same as the first one.

So, the possible histories of observations are

HHHHHHH….

Or

TTTTTTTTT…..

Again, predicting the future is easy! Once you know one state, you can predict all past and future states with ease!

Another example of a dynamical law is the following:

(Closed system with a coin)

Whatever the current state is, the next will be the opposite!

So if our object(Coin), shows heads, the next state is going to be tails.

T

H

Once again there are two possible histories:

HTHTHTHTHTH….

THTHTHTHTHT….

**Degrees of freedom:** The variables that describe a systeμ and that are also linked to the dynamical law of the system.

Our coin has one degree of freedom.

We denote the degrees of freedom with σ()

For example, our system has 2 degrees of freedom

σ = 1for H

σ =-1 for T

**When we are considering a continuous evolution in time, we symbolize time with t.**

**When we are considering a discrete evolution in time, we denote time with n.**

Since our system is stroboscopic and time evolves discretely(Number of observations), the **state** at  **time n** is described by **σ(n).**

**So far our equations are:**

**1st example σ(n) = σ(n+1)**

**2nd example σ(n+1) = -σ(n)**

The future depends completely on the initial state so these laws are **deterministic.**

**Cycle:** An endlessly repeat pattern.

* 1. **Rules that are not allowed**

In Classical Physics not all laws are allowed. A dynamical law needs to be both reversible and Deterministic.

**Reversible:** A law that is deterministic in the past and the future.

If in a system, we reverse the “arrows”, it should still be deterministic!

Meaning a system like this is not allowed:

T

H

In other words:

**The amount of arrows entering a state must be equal to the amount of arrows leaving the state, and must be equal to 1.**

**The Conservation of Information:** Every State has only 1 arrow in and only 1 arrow out.

* 1. **Dynamical Systems with infinite number of states.**

To describe a system of an infinite number of states, we will use **n** for time, since we talk bout discrete time steps, and  **N** for points on the track.

So a history of a system with infinite states and a certain law is a function **N(n)** telling the place along the track **N** at every time **n**.

Example:

…→0→1→2→….

This is a valid system since every state has one arrow pointing into it, and one arrow coming out of it!

We can express this rule as

**N(n+1) = N(n) + 1**

* 1. **Cycles and Conservation laws**

The State-Space can be divided into multiple cycles. When this happens, the system remains in the cycle it started in. Each cycle has its own dynamical rule, but they are all a part of the state-space.

**Conservation Law:** Whenever a dynamical law divides the state-space into separate cycles, there is a memory of which cycle they started in.

Each cycle has a numerical value called **Q**.

Whatever the value of  **Q**  is, it remains the same for all time, since the dynamical law does not allow jumping from one cycle to another.

So  **Q** is **conceived** through time.

* 1. **The Limits of precision**

**Resolving Power:** The power to distinguish the neighboring values of real numbers.

Real are infinitely dense, meaning that everyone one of them is arbitrary close to an infinite number of neighbors.

**Chaotic System:** A system in which the time over the system is predictable despite the resolving power limit.